Leptonic Decays of the W-Boson in a Strong Electromagnetic Field *

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Abstract

The probability of W-boson decay into a lepton and a neutrino, $W^{\pm} \to \ell^{\pm}\tilde{\nu}_{\ell}$, in a strong electromagnetic field is calculated. On the basis of the method for deriving exact solutions to relativistic wave equations for charged particles, an exact analytic expression is obtained for the partial decay width $\Gamma(\varkappa) = \Gamma(W^{\pm} \to \ell^{\pm}\tilde{\nu}_{\ell})$ at an arbitrary value of the external-field-strength parameter $\varkappa = eM_W^{-3}\sqrt{-(F_{\mu\nu}q^{\nu})^2}$. It is found that, in the region of comparatively weak fields, ($\varkappa \ll 1$) field-induced corrections to the standard decay width of the W-boson in a vacuum are about a few percent. As the external-field-strength parameter is increased, the partial width with respect to W-boson decay through the channel in question, $\Gamma(\varkappa)$, first decreases, the absolute minimum of $\Gamma_{\min} = 0.926 \cdot \Gamma(0)$ being reached at $\varkappa = 0.6116$. A further increase in the external-field strength leads to a monotonic growth of the decay width of the W-boson. In superstrong fields ($\varkappa \gg 1$), the partial width with respect to W-boson decay is greater than the corresponding partial width $\Gamma^{(0)}(W^{\pm} \to \ell^{\pm}\tilde{\nu}_{\ell})$ in a vacuum by a factor of a few tens.

1 INTRODUCTION

Presently, the Standard Model of electroweak interactions is the basis of our knowledge in the realms of elementary-particle physics. Many compelling pieces of evidence that the Standard Model describes correctly lepton and quark-interaction processes, which occur via the exchange of intermediate vector $W^\pm$ and Z^0 bosons, have been obtained over the past decade. The majority of experiments currently performed at the LEP and SLC electron-positron colliders are devoted to studying the properties of these particles, which mediate weak interactions [1]. Concurrently, the properties of W-bosons are being investigated at the Tevatron proton-antiproton accelerator. The accuracy reached in those experiments makes it possible to test the predictions of the Standard Model at the level of radiative corrections. For example, the upgrade of the LEP electron-positron collider, which is now referred to as LEP-2, enabled physicists working at CERN to observe, for the first time, the double production of W-bosons, $e^+e^- \to W^+W^-$ [2]. This reaction provides one of the most promising tools for precisely determining the W-boson mass ($M_W = 80.423 \pm 0.039$ GeV) and width ($\Gamma_W = 2.12 \pm 0.04$ GeV) [3]. Even at present, the errors in experimentally measuring the cross section $\sigma(e^+e^- \to W^+W^-)$ are as small as about one percent, whence it follows that theorists must calculate a number of $O(\alpha)$ radiative corrections to the tree diagrams for this process or radiative corrections of a still higher order. It should be noted that, because of W-boson instability, one actually has to deal with the more complicated multiparticle reaction $e^+e^- \to W^+W^- \to 4f$, where f stands for the fermion products of W-boson decay. Therefore, it is very difficult to perform an exact analytic calculation of all radiative corrections to the

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cross section for this process, and this will hardly be done in the near future.

In this connection, it is reasonable to discuss alternative methods for studying the properties of intermediate vector bosons. In this study, we aim at calculating the effect of strong electromagnetic fields on the leptonic mode of W-boson decay. The electromagnetic interactions of these particles, which are mediators of weak interactions, are the subject of special investigations [4].

The point is that the general form of the Lagrangian describing electromagnetic γWW interactions and satisfying the requirements of C and P invariance has not yet been obtained experimentally. From the theoretical point of view, it is quite admissible to extend the Standard Model by including in it new-physics effects that would generalize the minimal γWW vertex via the introduction of two dimensionless parameters k_γ and λ_γ [5],

$$\mathcal{L}_{\gamma WW} = -iek_\gamma F^{\mu\nu} W_\mu^+ W_\nu^- - ieW_{\mu\nu}^+ W^{-\mu} A^\nu + ieW_{\mu\nu}^- W^{+\mu} A^\nu +$$
$$+ie(\lambda_\gamma/M_W^2) W_{\mu\alpha}^+ W^{-\mu\beta} F^\alpha_{\ \beta}, \qquad (1)$$

where $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$, $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\quad A^\nu$ is the 4-potential of the electromagnetic field being considered. In the Standard Model of electroweak interactions due to Glashow, Weinberg, and Salam, these parameters have the following values at the tree level: $k_\gamma = 1$, $\lambda_\gamma = 0$. A precise experimental verification of these conditions must be performed by studying the reaction $e^+ e^- \rightarrow W^+ W^-$. However, this problem has yet to be solved conclusively.

In view of the aforesaid, the approach to studying the gauge structure of W-boson electromagnetic interactions on the basis of an analysis

of W-boson decays in an external field is of particular interest. In the present study, we calculate the probability of the reaction $W \to \ell\tilde{\nu}_\ell$ and the changes that this reaction induces in the total decay width Γ_W, relying on the method employing exact solutions to relativistic wave equations. It should be noted that precise measurements of the decay width of the W-bosons are of great interest both for theorists and for experimentalists. This is because all processes associated with the production of these particles at electron-positron colliders are investigated by analyzing the leptonic or hadronic products of W-boson decays. In addition, the W-boson width is used as one of the parameters that form a basis for calculating radiative corrections to electroweak processes occurring at energies in the vicinity of the W resonance; therefore, it is of paramount importance to have precise theoretical predictions for the width Γ_W.

One-loop radiative corrections to the decay width of the W-boson have already been calculated in the literature. In the approximation of massless fermions, they were obtained in a number of studies [6–11]. In that case, the main contribution comes from strong-interaction effects (about 4% with respect to the initial value of Γ_W) [12–14], whereas the corrections from electroweak processes are quite modest. It is noteworthy that new-physics phenomena (supersymmetry and so on) also make rather small contributions (see, for example, [15]). This is also corroborated by the calculations of the W-boson decay width within the two Higgs doublet model [16].

At the same time, the possibility of studying the properties of the W-bosons via changing external conditions under which their production occurs has not yet been explored. In high-energy physics, the method of channeling relativistic particles through single crystals, in which case such

particles are directed along the crystal axes and planes formed by the regular set of crystal-lattice atoms [17], has been known for a long time. The electric fields generated by the axes and planes of single crystals can reach formidable values (above 10^{10} V/m), extending over macroscopic distances. This changes substantially the physics of all processes in an external field in relation to the analogous phenomena in a vacuum. Thus, single crystals prove to be a unique testing ground where one can study reactions that become possible in the presence of a strong external electromagnetic field.

2 PROBABILITY OF W-BOSON DECAY IN AN EXTERNAL FIELD

In the leading order of perturbation theory, the matrix element for the reaction of W-boson decay to a lepton ℓ and a neutrino $\tilde{\nu}_\ell$ is given by

$$S_{fi} = \frac{ig}{2\sqrt{2}} \int d^4x \overline{\Psi}_\ell(x,p)\gamma^\mu(1+\gamma^5)\nu_\ell^c(x,p')W_\mu(x,q). \qquad (2)$$

Here, an external electromagnetic field is included through a specific choice of wave functions for the charged lepton ℓ and the W boson: $\Psi_\ell(x,p)$ and $W_\mu(x,q)$, respectively. Within this approach, one goes beyond ordinary perturbation theory, taking into account nonlinear and nonperturbative effects of an external field in the probability of the reaction under consideration. The explicit form of the wave functions for charged particles in an external field can be obtained by solving the corresponding differential equations that are determines by the Standard Model Lagrangian (see, for example [4, 18]). In the present study, we restrict our consideration to the case of a so-called crossed field, whose strength tensor satisfies

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the relations

$$F_{\mu\nu}F^{\mu\nu} = F_{\mu\nu}\tilde{F}^{\mu\nu} = 0. \tag{3}$$

Investigation of quantum processes in a crossed field is the simplest way to analyze the transformations of elementary particles in electromagnetic fields of arbitrary configuration. This is because all formulas obtained in the semiclassical approximation for the probabilities of processes that occur in a crossed field are also applicable to describing analogous processes in arbitrary constant electromagnetic fields. Thus, the crossed-field model appears to be the most universal in the region of relatively weak fields. At the same time, the wave functions for charged particles in a crossed field have the simplest form, and this renders relevant mathematical calculations much less cumbersome. In our case, the W-boson and the lepton (ℓ) wave function are expressed in terms of the external-crossed-field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as

$$\Psi_\ell(x,p) = \exp\left[-ipx - \frac{ie(pa)}{2(pFa)}(x^\mu F_{\mu\lambda}a^\lambda)^2 - \frac{ie^2}{6(pFa)}(x^\mu F_{\mu\lambda}a^\lambda)^3\right] \times$$
$$\times\left\{1 - \frac{e(xFa)}{4(pFa)}(F_{\mu\lambda}\gamma^\mu\gamma^\lambda)\right\}\frac{u(p)}{\sqrt{2p_0 V}}; \tag{4}$$

$$W_\mu(x,q) = \exp\left[-iqx - \frac{ie(qa)}{2(qFa)}(x^\mu F_{\mu\lambda}a^\lambda)^2 - \frac{ie^2}{6(qFa)}(x^\mu F_{\mu\lambda}a^\lambda)^3\right] \times$$
$$\times\left\{g_{\mu\nu} - \frac{e(xFa)}{(qFa)}F_{\mu\nu} + \frac{e^2(xFa)^2}{2(qFa)^2}(F_{\mu\lambda}a^\lambda)(F_{\nu\sigma}a^\sigma)\right\}\frac{v^\nu(q)}{\sqrt{2q_0 V}}. \tag{5}$$

The spin component of these wave functions, which are normalized to the three-dimensional volume V of the space, is determined by the Dirac bispinor $u(p)$ and the W-boson polarization 4-vector $v^\nu(q)$. Concurrently, it is assumed that the external-electromagnetic- field potential $A_\mu(x)$ is

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taken in the gauge

$$A_\mu(x) = -a_\mu(x^\alpha F_{\alpha\beta} a^\beta), \tag{6}$$

where the unit constant 4-vector a_μ satisfies the conditions

$$a_\mu a^\mu = -1, \qquad F_{\mu\nu} = (a_\mu F_{\nu\lambda} - a_\nu F_{\mu\lambda})a^\lambda. \tag{7}$$

Let us substitute the lepton and W-boson wave functions (4) and (5) into expression (2) for the S-matrix element and integrate $\mid S_{fi} \mid^2$ over the lepton and neutrino phase spaces. After some simple but cumbersome algebra, we obtain

$$\Gamma(W \to \ell\tilde{\nu}_\ell \mid \varkappa) = \frac{g^2 M_W}{48\pi^2} \int\limits_0^1 du \Big\{ \Big[1 - \frac{(m_\ell^2 + m_\nu^2)}{2M_W^2} - \frac{(m_\ell^2 - m_\nu^2)^2}{2M_W^4} \Big] \Phi_1(z) -$$
$$-2\varkappa^{2/3}\Big(\frac{u}{1-u}\Big)^{1/3}\Big[1 - 2u + 2u^2 + \frac{(m_\ell^2 + m_\nu^2)}{2M_W^2} \Big] \Phi'(z) \Big\}. \tag{8}$$

The partial decay width of the W-boson can be expressed in terms of the special mathematical functions $\Phi'(z)$ and $\Phi_1(z)$ (see Appendix), which depend on the argument

$$z = \frac{m_\ell^2 u + m_\nu^2(1-u) - M_W^2 u(1-u)}{M_W^2[\varkappa u^2(1-u)]^{2/3}}. \tag{9}$$

In the semiclassical approximation, the external-electromagnetic-field strength tensor appears in expression (8) for the partial decay width only in a combination with the W-boson energy-momentum 4-vector q^μ via the dimensionless parameter

$$\varkappa = \frac{e}{M_W^3}\sqrt{-(F_{\mu\nu}q^\nu)^2}. \tag{10}$$

The values of the parameter $\varkappa$ have a crucial effect on the W-boson width in an external field. From formula (8), one can see that, in the presence of an external electromagnetic environment, the rate of the reaction

$W \to \ell \tilde{\nu}_\ell$ becomes a dynamical characteristic that depends not only on the W-boson energy but also on external conditions under which this decay occurs. Therefore, we can no longer treat the W-boson width $\Gamma(W \to \ell \tilde{\nu}_\ell)$ as a constant since, in an external field, it becomes a function $\Gamma(\varkappa)$ of the parameter $\varkappa$.

Let us now consider asymptotic estimates of this function for various values of the external-electromagnetic-field strength. In the region of relatively weak fields that satisfy the constraint $\varkappa \ll m_\ell/M_W$, the partial decay width of the W-boson can be written as the sum of two terms; of these, one is coincident with the decay width in a vacuum, while the other is the correction induced by the electromagnetic-field effect:

$$\Gamma(W \to \ell \tilde{\nu}_\ell \mid \varkappa) = \Gamma^{(0)}(W \to \ell \tilde{\nu}_\ell) + \Delta\Gamma(\varkappa). \tag{11}$$

The quantity $\Gamma^{(0)}(W \to \ell \tilde{\nu}_\ell)$ is well known in the literature (see, for example, [2]). In the leading order of perturbation theory in the electroweak coupling constant g, it is given by

$$\Gamma^{(0)}(W \to \ell \tilde{\nu}_\ell) = \frac{g^2 M_W}{48\pi} \sqrt{\left[1 - \left(\frac{m_\ell + m_\nu}{M_W}\right)^2\right]\left[1 - \left(\frac{m_\ell - m_\nu}{M_W}\right)^2\right]} \times$$
$$\times \left[1 - \frac{(m_\ell^2 + m_\nu^2)}{2M_W^2} - \frac{(m_\ell^2 - m_\nu^2)^2}{2M_W^4}\right]. \tag{12}$$

As for the other term in expression (11) for the partial decay width, $\Delta\Gamma(\varkappa)$, its value depends nontrivially on the lepton and neutrino masses. In the case where the neutrino mass can be disregarded ($m_\nu = 0$), the effect of an external field on $\Delta\Gamma(\varkappa)$ is described by the relation

$$\Delta\Gamma(\varkappa) = -\frac{g^2 M_W}{48\pi} \cdot \frac{4}{3}\varkappa^2 \left[1 - \frac{13}{2}\left(\frac{m_\ell}{M_W}\right)^2 + 16\left(\frac{m_\ell}{M_W}\right)^4 - \frac{51}{4}\left(\frac{m_\ell}{M_W}\right)^6\right]. \tag{13}$$

This expression is the second term in the asymptotic expansion of the integral representation (8) for $\varkappa \to 0$. Here, the entire dependence on the

lepton mass m_ℓ is taken into account exactly, while the terms proportional to m_ν are discarded. The role of the effects induced by a nonzero neutrino mass is noticeable only in the region of very weak fields, $\varkappa \ll (m_\nu/M_W)^3$, in which case the expression for the field-induced correction $\Delta\Gamma(\varkappa)$ develops a characteristic oscillating term that modifies expression (13) as follows:

$$\Delta\Gamma(\varkappa) = \frac{g^2 M_W}{48\pi} \left[\frac{32}{\sqrt{6}} \varkappa \left(\frac{m_\nu}{M_W} \right)^3 \cos\left(\frac{\sqrt{3}}{8\varkappa} \frac{M_W}{m_\nu} \right) - \frac{4}{3} \varkappa^2 \right]. \qquad (14)$$

From the estimates obtained above, it can be seen that weak electromagnetic fields have virtually no effect on the decay width of the W-boson - that is, the corrections to the probability of the decay $W \to \ell\tilde{\nu}_\ell$ in a vacuum are within the errors of present-day experiments. However, this pessimistic conclusion is valid only for $\varkappa \ll m_\ell/M_W$. In the region of rather strong electromagnetic fields, the effects discussed here appear to be significant.

In view of this, we consider another limiting case, that of $\varkappa \gg m_\ell/M_W$. This relationship between the parameters makes it possible to disregard the lepton masses against the W-boson mass. The relative error of this approach does not exceed the level of corrections that are proportional to the ratio of the masses of these particles; that is,

$$\delta_\ell = \frac{m_\ell}{M_W} < 10^{-2}; \qquad \delta_\nu = \frac{m_\nu}{M_W} < 10^{-4}. \qquad (15)$$

By using the approximation $\delta_\ell = \delta_\nu = 0$, one can calculate analytically the W-boson decay width at any arbitrarily large value of the external-field-strength parameter $\varkappa$. The total result of our calculations is expressed in terms of the so-called Gi-function and its derivative (see Appendix). In order to render the ensuing exposition clearer, it is convenient to introduce the normalized partial decay width $R(\varkappa)$ that is defined as the ratio of the

decay width (8) in an external field to the analogous quantity in a vacuum; that is,

$$R(\varkappa) = \frac{\Gamma(W \to \ell\tilde{\nu}_\ell \mid \varkappa)}{\Gamma^{(0)}(W \to \ell\tilde{\nu}_\ell)}. \tag{16}$$

For the normalization factor, we chose the W-boson decay width (12) calculated at zero lepton and neutrino masses:

$$\Gamma^{(0)}(W \to \ell\tilde{\nu}_\ell) \simeq \frac{G_{\mathrm{F}} M_W^3}{6\pi\sqrt{2}} = 0.227 \;\; \mathrm{GeV}. \tag{17}$$

The ultimate formula determining the effect of electromagnetic fields on the leptonic modes of W-boson decay then takes the form

$$R(\varkappa) = \frac{4\pi}{81y}(19 - 2y^3)Gi'(y) - \frac{2\pi y}{81}(11 + 2y^3)Gi(y) + \frac{1}{81}(103 + 4y^3), \tag{18}$$

where the argument of the Gi-functions is related to the parameter $\varkappa$ by the equation $y = \varkappa^{-2/3}$. This expression is very convenient for computer calculations of the rate of W-boson decay in an external field. The results of the present numerical analysis that was based on the "Mathematica" system are displayed in Fig.1. The graph there represents the quantity $R(\varkappa)$ (16) as a function of the electromagnetic-field strength $(\varkappa)$. One can see that, as the parameter $\varkappa$ increases, the partial decay width of the W-boson gradually decreases. In the intermediate region $\delta_\ell \ll \varkappa \ll 1$, this decay width is well described by the asymptotic formula

$$\Gamma(W \to \ell\tilde{\nu}_\ell) = \frac{g^2 M_W}{48\pi}\left(1 - \frac{8}{3}\varkappa^2 - \frac{304}{3}\varkappa^4 - \frac{4}{\sqrt{3}}\frac{\delta_\ell^3}{\varkappa}\right). \tag{19}$$

It is noteworthy that the numerical coefficient of the first correction to the W-boson decay width in a vacuum, $(8/3)\varkappa^2$, is precisely two times as great as the analogous coefficient of $\varkappa^2$ in the region $\varkappa \ll \delta_\ell$ [see (13), (14)]. This indicates that the partial decay width of the W-boson in an external field takes a minimum value in the region around $\varkappa \sim 1$. A computer
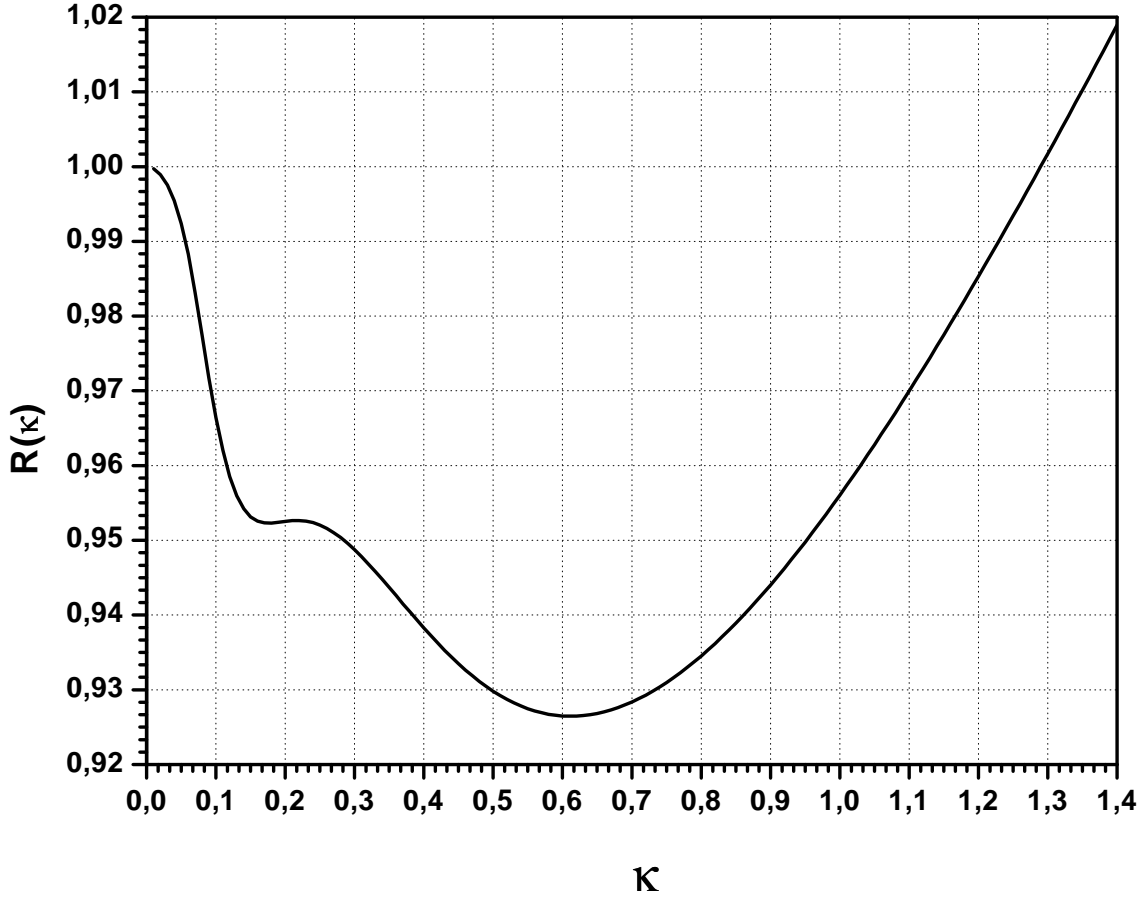
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Figure 1: Relative variations in the leptonic-decay width of the W-boson in weak electromagnetic fields.

calculation shows that the quantity $R(\varkappa)$ reaches the absolute minimum of $R_{\min} = 0.926$ at $\varkappa_{\min} = 0.6116$. Thus, we can state that, in weak fields, the deviation of the partial decay width of the W-boson from that in a vacuum does not exceed 7.4%. A further increase in the external-field strength leads to a monotonic growth of the probability of W-boson decay, so that, at $\varkappa > 1.3$, the decays to a lepton and a neutrino occur faster than those in a vacuum $(R(\varkappa) > 1)$. In this region of relatively strong fields, the mean lifetime of the W-boson decreases sizably, which is illustrated
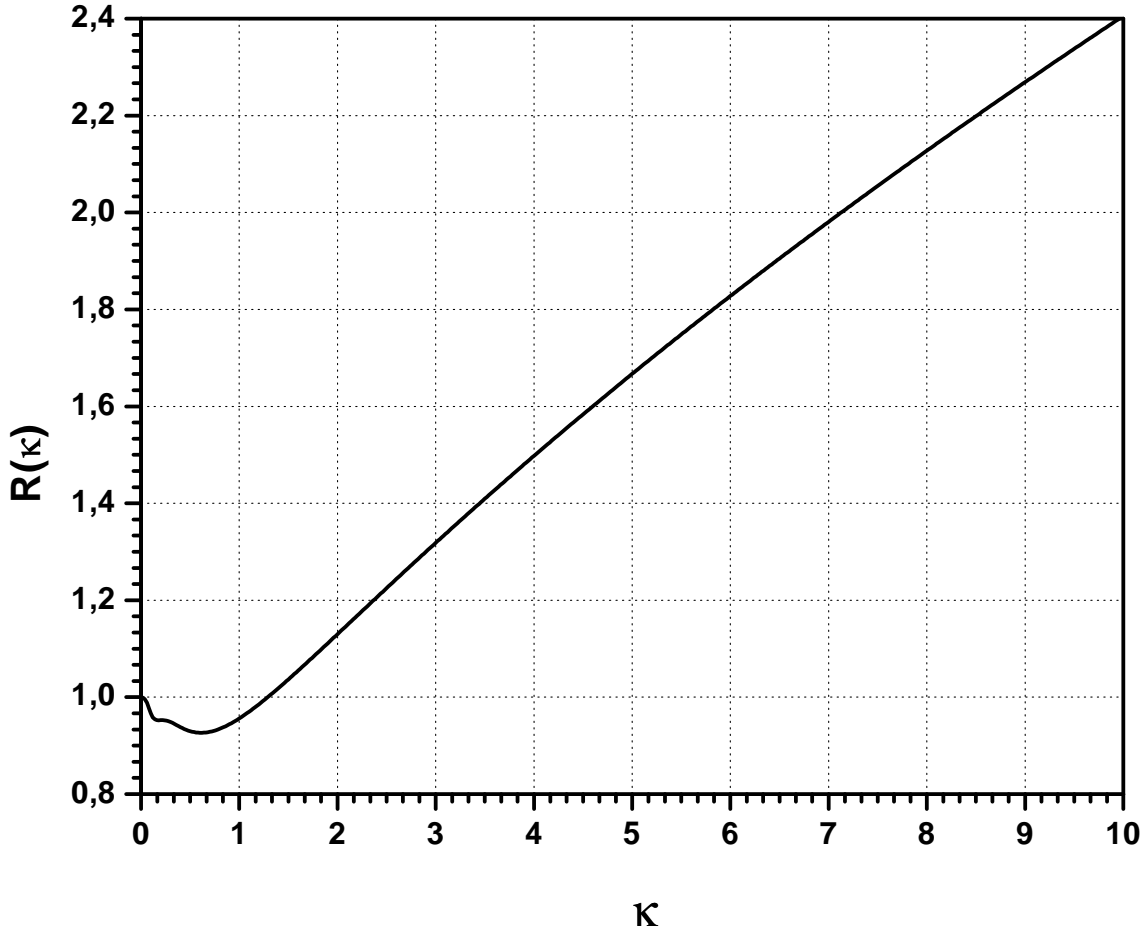
Figure 2: Effect of strong electromagnetic fields on the leptonic mode of W-boson decay.

by the graph in Fig.2. One can see that, even at $\varkappa = 7$, the dynamic al width of the W-boson becomes two times larger than the static vacuum value in (12). In superstrong fields ($\varkappa \gg 1$), the partial decay width of the W-boson can be estimated with the aid of the asymptotic expression

$$\Gamma(W \to \ell\tilde{\nu}_\ell) = \frac{g^2 M_W}{48\pi}\left\{\frac{38}{243}\Gamma(2/3)(3\varkappa)^{2/3} + \frac{1}{3} + \frac{8}{81}\frac{\Gamma(1/3)}{(3\varkappa)^{2/3}}\right\}. \qquad (20)$$

3 CONCLUSION

The effect of strong electromagnetic fields on the leptonic mode of W-boson decay has been investigated. We have revealed that, in an external field, the partial width with respect to the decay $W \to \ell\tilde{\nu}_\ell$ is a nonmonotonic function of the field-strength parameter $\varkappa$ (10). In particular, there is a domain of field-strength values where the decays of the W-boson occur somewhat more slowly than those in a vacuum. This fact has a significant effect on the total decay width of the W-boson, since, in the approximation of massless leptons and quarks, this width is known to be related to the partial width with respect to the leptonic decays considered here by the equation

$$\Gamma_W \simeq (N_\ell + N_q N_c) \cdot \Gamma(W \to \ell\tilde{\nu}_\ell) = 12 \cdot \Gamma(W \to \ell\tilde{\nu}_\ell), \qquad (21)$$

where $N_\ell = N_q = 3$ are the numbers of the lepton and quark generations and $N_c = 3$ is the total number of color quarks. Thus, one can state that, in relatively weak electromagnetic fields, the maximum deviation of the total width of the W-boson from its vacuum value Γ_W is 7%. As for the region of strong fields ($\varkappa \geq 1$), a stable trend toward an increase in the rate of W-boson decays is observed here, which reduces their lifetime to a still greater extent. It is noteworthy that a similar nonmonotonic dependence on the external-field-strength parameter is observed in the decay of the scalar pion to a lepton pair, $\pi \to \ell\tilde{\nu}_\ell$ [19] This circumstance is explained by the similarity of kinematical conditions under the which the decays of massive charged particles proceed in a crossed field and by the fact that the reactions $W \to \ell\tilde{\nu}_\ell$ and $\pi \to \ell\tilde{\nu}_\ell$ are both energetically allowed; therefore, they are possible even in the absence of external fields. At the same time, it should be noted that the electromagnetic interac-

tions of the W-bosons are much more complicated than the interactions of scalar pions, this leading to a number of interesting phenomena. It is well known (see, for example, [20]) that the energy spectrum of W-bosons in a superstrong electromagnetic field involves the so-called tachyon mode, which is due to their anomalous magnetic moment $\Delta\mu_W = ek_\gamma/2M_W$. This results in that the W-boson vacuum becomes unstable within perturbation theory, so that there arises, in super-strong fields, the possibility of a phase transition to a new ground state, this phase transition being accompanied by the restoration of $SU(2)$-symmetry, which is spontaneously broken under ordinary conditions [21–24]. The trend toward the rearrangement of the W-boson vacuum manifests itself in a singular behavior of many physical quantities in the vicinity of the critical external-field-strength value of $F_{\rm cr} = M_W^2/e \simeq 1.093 \times 10^{24}$ G. For example, the anomalous magnetic moment of Dirac neutrinos that is due to the virtual production of W-bosons has a logarithmic singularity in a superstrong field for $F = \sqrt{F_{\mu\nu}F^{\mu\nu}/2} \to F_{\rm cr}$ [25]. Unfortunately, the crossed-field model, which was used in the present study, is inadequate to the problem of calculating the behavior of the W-boson decay width at external-field strengths close to the critical value $F_{\rm cr}$. In order to solve this problem, it is necessary to go beyond the semiclassical approximation (3) and to employ more complicated wave functions for charged particles. At any rate, it seems that this problem is of particular interest and that it deserves a dedicated investigation.

Yet another interesting result obtained in the present study is that which concerns the effect of a nonzero mass of Dirac neutrinos on the W-boson decay width in weak electromagnetic fields. In the case of $m_\nu \neq 0$, it has been found that the correction $\Delta\Gamma(\varkappa)$ to the vacuum decay width

develops a nontrivial oscillating term (14), which can serve as some kind of indication that massive neutrinos exist. It should be noted that oscillations of the probabilities of quantum processes in an external field emerge for a number of reactions allowed in a vacuum if the participant particles have a nonzero rest mass. For example, similar oscillations arise in the cross section for the process $\gamma e \rightarrow W \nu_e$ if the respective calculations take into account effects of a nonzero neutrino mass [26].

APPENDIX

In the present study, we have employed special mathematical functions generically termed Airy functions. A somewhat different notation is used for Airy functions in the physical and in the mathematical literature. Mathematicians describe Airy functions with the aid of the symbols $Ai(z), Bi(z)$, and $Gi(z)$, which are related to our notation as follows:

$$\Phi(z) = \pi Ai(z), \qquad \Phi'(z) = \pi Ai'(z), \tag{A.1}$$

$$\Phi_1(z) = \pi^2 [Ai(z)Gi'(z) - Ai'(z)Gi(z)]. \tag{A.2}$$

Airy functions are particular solutions to the second-order linear differential equations

$$Ai''(z) - zAi(z) = 0, \qquad Gi''(z) - zGi(z) = -\frac{1}{\pi}, \tag{A.3}$$

These solutions can be represented in the form of improper integrals of trigonometric functions,

$$Ai(z) = \frac{1}{\pi} \int_0^\infty dt \cos(zt + t^3/3), \tag{A.4}$$

$$Gi(z) = \frac{1}{\pi} \int_0^\infty dt \sin(zt + t^3/3). \tag{A.5}$$

The properties of the Airy functions are well known, and a compendium of these properties can be found in mathematical handbooks (see, for example, [27]).

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